

ÉRETTSÉGI VIZSGA • 2009. május 5.

**MATEMATIKA
ANGOL NYELVEN**

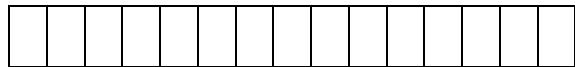
**EMELT SZINTŰ
ÍRÁSBELI VIZSGA**

2009. május 5. 8:00

Az írásbeli vizsga időtartama: 240 perc

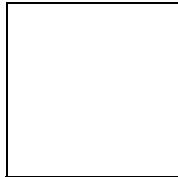
| Pótlapok száma | |
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| Tisztázati | |
| Piszkozati | |

**OKTATÁSI ÉS KULTURÁLIS
MINISZTÉRIUM**



Important information

1. The exam is 240 minutes long, after that you should stop working.
2. You may solve the problems in any order.
3. In Section **II**, you are only required to solve four out of the five problems. **Please remember to enter the number of the question you have not attempted into the empty square below.** Should there arise any ambiguity for the examiner as for the question not be marked, it is question no. 9 that will not going to be assessed.



4. You may work with any calculator as long as it is not capable of storing and displaying textual information and you may also consult any type of four digit mathematical table. The use of any other kind of electronic device or written source is forbidden.
5. **Remember to show your reasoning, because a major part of the score is given for this component of your work.**
6. **Remember to outline the substantial calculations.**
7. When you refer to a theorem that has been covered at school and has a common name (e.g. Pithagoras' theorem, sine rule, etc.) you are not expected be state it meticulously; it is usually sufficient to put the name of the theorem. Any reference to any other theorem, however, can be accepted only if it is stated exactly with all the conditions (proof is not required) and you explain how it applies in the given situation.
8. Remember to answer each question (i.e. communicating the result) also in textual form.
9. You are supposed to work in pen; diagrams, however, may also be drawn in pencil. Anything written in pencil outside the diagrams cannot be evaluated by the examiner. Any solution or some part of a solution that is crossed out will not be marked.
10. There is only one solution will be marked for every question. If you attempt a question more than once then you should **clearly indicate** the one to be marked.
11. Please, do not write anything in the shaded rectangular areas.

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I.

- 1.** A survey was investigating in the average time spent on studying in the course of a week in a class of 26 students. The results are summarized in the following table.

| | | | | | | | | |
|-------------------------|---|---|---|---|---|---|---|----|
| Hours spent on studying | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Number of students | 6 | 3 | 1 | 2 | 0 | 5 | 5 | 4 |

- a)** Calculate the average number of hours a student spent on studying in a week in this class. Determine further statistics (i.e. mode and median) of the average weekly studying hours.
b) Prepare a bar chart based on the data of the above table.

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| a) | 7 points | |
| b) | 3 points | |

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- 2.** A coffee trading company is delivering mixtures from two kinds of coffee. If a mixture is composed from 20 kg of A-type coffee and 30 kg of B-type coffee then the price of the mixture is 1860 Ft/kg.

If, however, the mixture is composed of 30 kg of type-A coffee and 20 kg of B-type coffee then its price is 1740 Ft/kg.

- a) What is the price of 1 kg coffee of types A and B, respectively?
- b) The company wants to produce 60 kg of mixture for 2000 Ft/kg. How many kilograms of coffee do they have to use from the types A and B, respectively?

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| a) | 10 points | |
| b) | 4 points | |

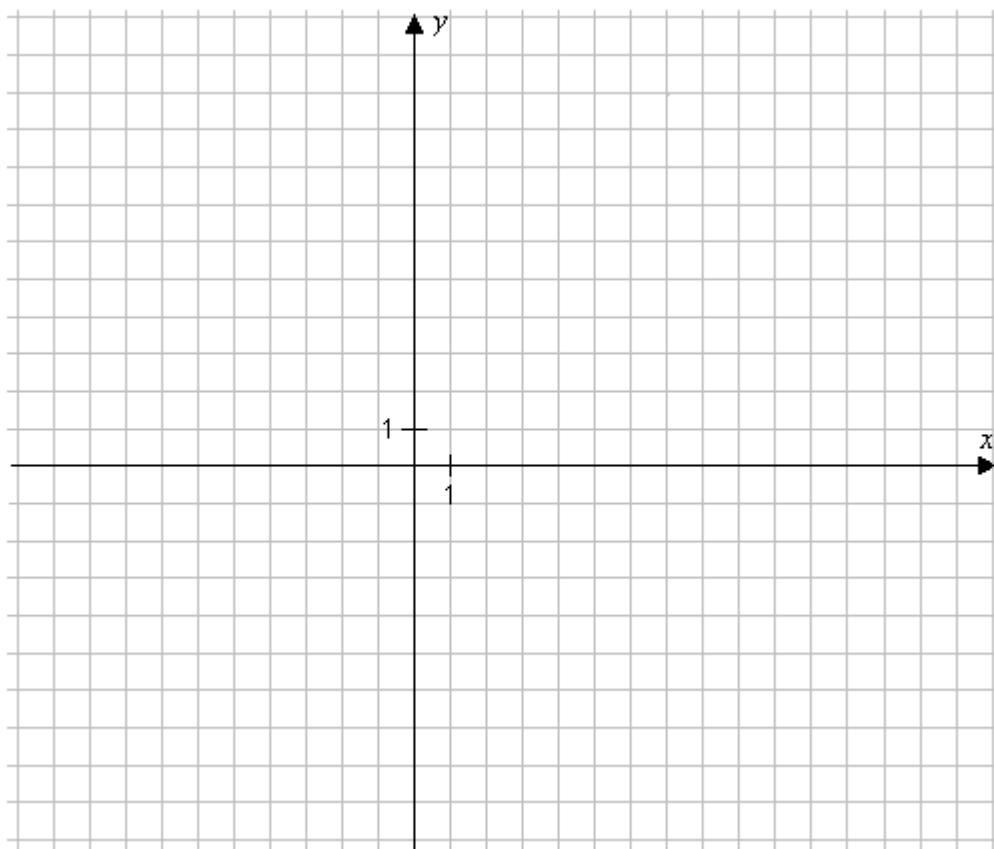
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3. The function $x \mapsto 2x^2 - 4x - 6$ is defined on the set of real numbers.

- a) Calculate the zeroes of the function, its minimum value and also the value of x where this minimum is attained.
b) Sketch the graph of the function on the interval $[-2; 4]$.
c) Determine the coordinates of the focus of the parabola of equation $y = 2x^2 - 4x - 6$.

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| a) | 6 points | |
| b) | 3 points | |
| c) | 4 points | |



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4. Solve the following inequality on the set of real numbers.

$$\sqrt{x^2 - 3x} \cdot \log_{0.1}(x + 2) < 0 .$$

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II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

- 5.** The first three terms of a geometric progression of positive terms are a , b , c . If the first two terms are left unchanged and the third term is reduced by $(a + 2b)$, then the first three terms of an arithmetic progression are obtained. The numbers a , $b + 9$, c in this order are also three consecutive terms of an arithmetic progression.

Determine the numbers a , b and c .

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| 16 points | |
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You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

6. a) How many six-digit numbers can be made out of the digits 0, 1, 2, 3, 4, 5 if the same digit may be used more than once?
b) How many numbers are there among the above six-digit numbers that consist of different digits and are divisible by five?
c) How many six-digit numbers can be made out of the digits 0, 1, 2, 3, 4, 5 in which there is at least one repeating digit? (At least one digit occurs at least twice.)

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| a) | 3 points | |
| b) | 6 points | |
| c) | 7 points | |

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You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

7. András and Bálint went hiking at night. When they reached an open flat area, they could see distant fireworks displays in two different directions at the same time. In order to determine the distance between the two fireworks sites, they measured that the noise of the explosions reached them 18 seconds after the light flashes from one direction, and 14 seconds after the light flashes from the other direction. They assumed that the speed of sound was $340 \frac{\text{m}}{\text{s}}$, and they neglected the time taken by light propagation. Then, since they did not have a protractor at hand, András took 32 steps in the direction of one fireworks site, and Bálint also took 32 steps towards the other. (Taking care that they use steps of the same length.) They measured that they got 60 steps away from each other.
- a) Using the data measured by András and Bálint, calculate the distance between the two fireworks sites to the nearest kilometre.

The hikers covered half the distance at an average speed of $2 \frac{\text{km}}{\text{h}}$ and the other half at an average speed of $5 \frac{\text{km}}{\text{h}}$.

- b) What was their average speed for the whole distance?

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| a) | 10 points | |
| b) | 6 points | |

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You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

8. The diameter of the base of a right circular cone is 10 cm, and its slant height is 13 cm. Inscribe a right cylinder of maximum volume in the cone, such that the cone and the cylinder have a common axis of symmetry, and the base of the cylinder lies on the base of the cone.

What is the radius of the inscribed cylinder?

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You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

- 9.** At a certain music school one can learn to play three instruments: the piano, the guitar and the saxophone. Last year, 18 students enrolled in the courses. Each of them learnt to play one or two instruments, no one learnt all three. There were fifteen piano students, eight guitar students and seven saxophone students.

- a) How many of them learnt to play exactly two instruments?

There was no student in the school who learnt to play both the guitar and the saxophone. Among those who only learnt to play one instrument, there were twice as many saxophone students as guitar students.

- b) How many students learnt to play both the piano and the guitar? How many learnt to play both the piano and the saxophone?

- c) Two tickets to the same concert were awarded to two randomly selected students of the music school. What is the probability that either both of them learnt to play the saxophone or they both learnt to play the guitar?

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| a) | 3 points | |
| b) | 7 points | |
| c) | 6 points | |

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| | number of question | maximal score | score | maximal score | score |
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| PART I | 1. | 10 | | 51 | |
| | 2. | 14 | | | |
| | 3. | 13 | | | |
| | 4. | 14 | | | |
| PART II. | | 16 | | 64 | |
| | | 16 | | | |
| | | 16 | | | |
| | | 16 | | | |
| | ← problem not chosen | | | | |
| TOTAL | | | 115 | | |

date

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| | score attained (elért pontszám) | score input for program (programba beírt pontszám) |
|---------------------|---------------------------------------|---|
| Paper I/(I. rész) | | |
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